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Non-commutative tachyons

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Abstract: When unstable Dp-branes in type-II string theory are placed in a B-field, the resulting tachyonic world-volume theory becomes non commutative. We argue that for large noncommutativity parameter, condensation of the tachyon as a non-commutative soliton leads to new decay modes of the Dp-brane into (p−2)-brane configurations, which we interpret as suitably smeared BPS D(p − 1)-branes. Some of these configurations are metastable. We discuss various generalizations of this decay process.

Keywords: Solitons Monopoles and Instantons, Superstrings and Heterotic Strings, D-branes, Non-Commutative Geometry.

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Contents

1. Introduction

The decay modes of unstable D-branes and brane-antibrane pairs have been extensively studied in the last couple of years (for reviews see for example refs. [1, 2, 3]). Some of the most basic decay modes found so far are: annihilation into vacuum [4]–[7], annihilation via kink condensation into a brane of codimension one [8]–[11], and annihilation via vortex condensation to a brane of codimension two [9, 12]. Condensation of higher-codimension topological solitons has also been studied [12, 11]. Some of these decay modes correspond to stable solitons, and in this case the end-products are stable branes, while in other cases the decay modes correspond to unstable solitons and lead to unstable branes. More exotic decay modes are possible when unstable branes or brane-antibrane pairs are either wrapped on homology cycles of non-trivial manifolds [13, 14, 15] or suspended between other branes in brane constructions [16].

In this note, we identify a class of novel processes in which unstable D-branes decay via condensation of a non-commutative soliton. Such solitons [17] are finite-energy classical solutions in non-commutative scalar field theories with large non-commutativity parameter θ_{ij}. Physically, this situation can be realised by turning on a suitable constant NS-NS B-field along the brane world-volume.

Since non-commutative solitons carry no topological charge and are sometimes metastable, the same will be true of the decay products of an unstable brane when such a soliton condenses on its worldvolume. We will interpret these decay products by examining what Ramond-Ramond charge they carry locally.

While this work was nearing completion, we became aware of a forthcoming paper [18] which addresses related questions, mainly in the context of the bosonic string.

2. Non-commutative tachyons and brane decay

Consider an infinitely extended, unstable D2-brane in type-IIB string theory. We place it in a $B_{\mathrm{NS,NS}}$ field with a non-zero component only along the brane world-volume directions, namely B_{12}. Consider the brane world-volume theory. The light modes are a real tachyon, a massless U(1) gauge field and some massless fermions.

Let us set all fields on the brane, except the tachyon, to 0. The spacetime backgrounds are also as simple as possible: a flat metric and the constant B_{12} field. Following arguments in refs. [19, 20], the effect of the B-field can be described by making the tachyonic field theory on the D2-brane world-volume into a non-commutative field theory with $*$ product given by

$$f(x) * g(x) = e^{\frac{i}{2}\theta(\partial_1\partial_2' - \partial_2\partial_1')}\, f(x)\; g\left(x'\right)\big|_{x=x'}\,, \tag{2.1}$$

where the noncommutativity parameter θ_{ij}, given in terms of B by [19, 20]

$$\theta_{ij} = -\left(2\pi\,\alpha'\right)^2 \left(\frac{1}{g + 2\pi\,\alpha'\,B}\,B\,\frac{1}{g - 2\pi\,\alpha'\,B}\right)_{ij} \tag{2.2}$$

has the single component $\theta_{12} = \theta$.

Let $V(T)$ denote the universal tachyon potential [21] on an unstable D-brane. The action for such a non-BPS D-brane is given by [22, 23, 24]:

$$\mu_p^{(u)} \int d^{p+1}\,\sigma\, V(T) \sqrt{-\det\left(g_{\mu\nu} + 2\pi\,\alpha'\,\mathcal{F}_{\mu\nu} + 2\pi\,\alpha'\,\partial_\mu T\,\partial_\nu T\right)}\,, \tag{2.3}$$

where $g_{\mu\nu}$ is the closed string metric and $\mathcal{F}_{\mu\nu}$ is the linear combination of $F_{\mu\nu}$ and $B_{\mu\nu}$. We claim that in the presence of a B-field, the tachyonic part of the D2-brane worldvolume action is of the form

$$S = \int d^2x\, dt\, \Big(\partial^\mu T\,\partial_\mu T - V(*T)\Big)\,, \tag{2.4}$$

where the notation $V(*T)$ represents the universal tachyon potential with all products replaced by $*$ products.

To argue this, note that the replacement of $V(T)$ by $V(*T)$ apparently violates universality of the tachyon potential. However, precisely because this universality holds only in the zero-momentum sector [21], we can expect a violation of universality that vanishes at zero-momentum. This is true of the $*$ product, which reduces to the ordinary product on constant functions. Thus, noncommutativity and universality of the tachyon potential together amount to a derivation of the above action.[1]

For unstable D-branes in type-II string theory, $V(T)$ is known to be an even function of T and is believed to have a double-well shape. Let it take its minima at $T = \pm T_0$. At the minimum, the negative potential energy localized on the brane

[1] We are grateful to K. Hori for discussions on this point.

should cancel the unstable D2-brane tension $\mu_2^{(u)}$, consistent with the decay of the D2-brane into vacuum. This gives rise to the beautiful equation [1]:

$$\mu_2^{(u)} + V\,(T = T_0) = 0\,. \tag{2.5}$$

Besides decay into the vacuum, the simplest decay mode of the unstable D2-brane is via kink condensation in this double-well potential, leading to a stable D-string. The D-string charge arises from the Chern-Simons coupling [11, 22, 25]:

$$\frac{1}{2\,T_0}\int dT \wedge B_{\rm RR}\,, \tag{2.6}$$

where $B_{\rm RR}$ is the Ramond-Ramond 2-form in type IIB. For a kink solution along say x^2, we have $\int dx^2\,\partial_2 T = 2\,T_0$ and we are left with unit coupling to $B_{\rm RR}$ and hence unit D-string charge. An anti-kink would produce an anti-D-string.

We work in the limit of large θ. It is possible to achieve this limit, while keeping the open string metric [20] fixed, by a suitable choice of scaling for α', B and the closed string metric g, for example, the limit

$$\alpha' \longrightarrow \sqrt{\epsilon}\,, \qquad B \longrightarrow \sqrt{\epsilon}\,, \qquad g \longrightarrow \epsilon^2 \tag{2.7}$$

with $\epsilon \to 0$. Following ref. [17], it is convenient to scale the coordinates by $x^i \to \sqrt{\theta}\,x^i$, and one ends up with the action

$$S = \int d^2x\,dt\left(\partial^\mu T\,\partial_\mu T - \theta\,V(*T)\right), \tag{2.8}$$

where now the $*$ product is as in eq. (2.1) but with $\theta = 1$.

At infinite θ, we can look for classical solutions by just solving $\partial V(*T)/\partial T = 0$. For a fairly arbitrary tachyon potential (but in particular, without a linear term), an infinite set of non-constant solitonic solutions of this equation was found in ref. [17]. In particular, one solution is provided by starting with the function $\phi_0(x) \equiv 2\,e^{-r^2}$ and writing

$$T(x) = \lambda_i\,\phi_0(x)\,, \tag{2.9}$$

where λ_i is any non-zero minimum of the ordinary potential. This works by virtue of the fact that (with $\theta = 1$), we have $\phi_0 * \phi_0 = \phi_0$.

Note that this soliton decays to zero at infinity. Although this was not explicitly stated, the analysis and examples of ref. [17] dealt entirely with potentials having a quadratic minimum at the origin, hence the soliton does in fact tend asymptotically to the vacuum. Our case is just the opposite: a tachyonic potential has a quadratic maximum at the origin. One can of course go from one to the other by shifting $T(x)$ by a specific constant, though general shifts of $T(x)$ are not allowed — since they introduce a linear term in $V(T)$, which in turn invalidates the solutions of ref. [17]. As we will now see, a small modification of the example above by such a constant shift will give rise to an interesting physical phenomenon with unstable branes. Suppose we first take the classical solution:

$$T(x) = T_0\,(1 - \phi_0(x))\,. \tag{2.10}$$

Note that, like $\phi_0(x)$, $(1-\phi_0(x))$ also squares to itself under the $*$ product. However, it varies from $+1$ at spatial infinity to -1 at the origin. Thus the configuration $T(x)$ above is a soliton that interpolates from T_0 at spatial infinity to $-T_0$ at the origin. One can of course take the negative of this solution and it interpolates in the opposite way.

A key insight into the physical meaning of this process can be obtained by looking at the total energy of this soliton. The energy density of the soliton is easily evaluated using $(1-\phi_0) * (1-\phi_0) = (1-\phi_0)$:

$$V\Big(T_0\,(1-\phi_0(x))\Big) = (1-\phi_0(x))\,V(T_0)\,. \tag{2.11}$$

Hence the total energy density on the D2-brane, with the non-commutative soliton excited, is

$$\theta\left(\mu_2^{(u)} + (1-\phi_0(x))\,V\,(T_0)\right). \tag{2.12}$$

Using Sen's conjecture for unstable brane decay, eq. (2.5), this works out to be $\theta\,\phi_0(x)\,\mu_2^{(u)}$. This has to be interpreted as the energy of the decay product. We see that this decay product is an object whose energy is localised very close to the origin, so it can be considered an exotic 0-brane. It is planar and its energy distribution falls off exponentially with the distance away from the origin. Indeed, it is a D0-brane in a sense, since a string that was ending on the original D2-brane can continue to end on regions where there is finite D2-brane tension. Hence we interpret the non-commutative soliton above as describing the decay of a planar unstable D2-brane into a localised configuration at the origin. We will argue in the next section that this configuration is really a smeared D-string.

It is noteworthy that the energy of the non-commutative soliton can be calculated exactly without a detailed knowledge of the tachyon potential. This is due to the fact, pointed out in ref. [17], that the soliton solution depends on no details of the potential except its value at the minimum. Remarkably, at the present stage of understanding of string theory, this is the only thing about the tachyon potential on D-branes that we *do* know reliably, thanks to eq. (2.5).

It is clearly important to analyse various properties of this D0-brane including its stability (in the presence of large noncommutativity). The above solution will turn out to be classically unstable, as we will argue in section 4.

There is, in fact, another classical solution which is most interesting from the point of view of stability. This is given by:

$$T(r) = T_0\,(1 - 2\phi_0(r))\,. \tag{2.13}$$

That these are classical solutions follows from the fact that they can equivalently be written

$$T(r) = T_0\,(1 - \phi_0(r)) + (-T_0)\,\phi_0(r) \tag{2.14}$$

in which form it is evident that they are the superposition[2] of two non-commutative solitons, one of which asymptotes to 0 and the other to T_0. Another way of seeing that these are solutions is that although the function $(1 - 2\phi_0(r))$ does not square to itself under the $*$ product (rather it squares to 1), its *odd* powers are all equal to itself, and that is sufficient because the tachyon potential is even.

At the origin this solution tends to $-3\,T_0$. We will see later that this solution is metastable. The energy of this solution is

$$V\Big(T_0\,(1 - 2\,\phi_0(r))\Big) = (1 - \phi_0(r))\,V(T_0) + \phi_0(r)\,V\,(-T_0) = V\,(T_0)\,. \tag{2.15}$$

This is degenerate with the energy of the vacuum solution. Thus, in this decay mode the original D2-brane has decayed into a non-trivial configuration that has the same energy as the vacuum.

Before turning to the issue of stability, we look at some generalizations. A complete set of radially symmetric solutions to the equation $\phi * \phi = \phi$ was worked out in [17].[3] They satisfy the relation:

$$\phi_n(r) * \phi_m(r) = \delta_{m\,n}\,\phi_n(r)\,, \qquad \sum_{n=0}^{\infty} \phi_n(r) = 1\,. \tag{2.16}$$

These functions may be found as follows (the following constitutes a quick alternate derivation to the one in ref. [17]). Define the generating function

$$\phi(r, z) = \sum_{n=0}^{\infty} \phi_n(r)\,z^n \tag{2.17}$$

in terms of which the above equations amount to

$$\phi(r, z) * \phi(r, z') = \phi(r, zz')\,, \qquad \phi(r, z = 1) = 1\,. \tag{2.18}$$

The first of these equations may be solved by going to Fourier space in x (recall that above, $r = |x|$). One finds that, with a radially symmetric ansatz, the solution is

[2] Normally, one would not expect superpositions of solutions to be solutions in a non-linear theory. However, the orthogonality properties of the ϕ_n under the star product allow us to superpose orthogonal solutions.

[3] The equation $\phi * \phi = \phi$ has a long history, starting with work of Takabayasi [26] in 1954, where it appears as the normalization condition for pure-state Wigner functions. The equation and its solutions have been studied in some interesting recent works [27, 28]. We are grateful to Spenta Wadia and Cosmas Zachos for communicating this to us.

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gaussian in Fourier space and hence also in coordinate space. Inserting this ansatz and solving the above equations, one ends up with the generating function:

$$\phi(r,z) = \frac{2}{1+z}\, e^{\frac{z-1}{z+1}r^2}\,. \tag{2.19}$$

Expanding in powers of z one finds:

$$\phi_n(r) = 2\,(-1)^n\, e^{-r^2}\, L_n(2\,r^2)\,, \tag{2.20}$$

where L_n are the Laguerre polynomials, in agreement with the solution in ref. [17].

We can now write the obvious generalization of eq. (2.10) , $T_n(r) = T_0(1-\phi_n(r))$, noting that $(1-\phi_n(r))$ squares to itself under the star product. This soliton is also a D0-brane, but is $\sqrt{n}$ times larger than the original one. While for even n the solution interpolates between one vacuum $T = T_0$ at infinity to the other vacuum $T = -T_0$ at the origin, for odd n the situation is quite different. From $\phi_n(r=0) = 2\,(-1)^n$, we see that the solutions for odd n interpolate between the vacuum $T = T_0$ at infinity and a non-vacuum configuration $T = 3\,T_0$ at the origin. There is, of course, no requirement that the soliton should go to a vacuum at the origin.

It is also easy to see that $T(r) = T_0(1-f(r))$, with

$$f(r) = \sum_{i\in I} \phi_i(r)\,, \tag{2.21}$$

looseness=1 where the sum runs over a finite index set I of the ϕ_i's, each counted exactly once, is also a classical solution, since such a function $f(r)$ also squares to itself under the star product. Functions of the form $(1-f(r))$ with $f(r)$ as above are, in fact, the most general functions that square to themselves and have the asymptotic behaviour of a solitonic solution for a tachyonic potential, but as we will see in a moment, these are not the most general extrema of the non-commutative tachyon potential.

It is interesting to note the relation

$$(1-\phi_n) = \sum_{m\neq n} \phi_m \tag{2.22}$$

so that the above solitons may be regarded as the superposition of an infinite set of the non-commutative solitons of [17]. Note that since ϕ_n vanishes at infinity, the asymptotics of the functions $(1-\phi_n)$ and ϕ_n are very different. There is, however, no contradiction with (2.22) because the r.h.s. is an infinite sum.

It is also easy to write down a generalization of eq. (2.13). Given that the tachyon potential is an even function, we can find a class of solutions with zero total energy using functions $f(r)$ that satisfy $f(r) * f(r) = 1$. Using any such function we can define a solution $T(r) = T_0 f(r)$ that has $V(T(r)) = V(T_0)$ and hence vanishing total energy.

Expanding such functions over the $\phi_n(r)$ we have:

$$f(r) = \left(1 + \sum_{i \in I} \lambda_i \phi_i(r)\right), \tag{2.23}$$

where again I is a finite index set of distinct elements. One sees that

$$f(r) * f(r) = 1 + \sum_{i \in I} \lambda_i (\lambda_i + 2) \phi_i(r) \tag{2.24}$$

from which it is clear that each λ must be equal to 0 or -2. Thus the classical solutions

$$T(r) = T_0 \left(1 - 2 \sum_{i \in I} \phi_i(r)\right) \tag{2.25}$$

all have zero energy and will turn out to be metastable.

Finally, we can superpose the two kinds of solutions above to obtain the most general solitonic solution (upto an overall choice of sign, of course) for a double well potential

$$T_r = T_0 \left(1 - \sum_{i \in I} \phi_i\right) + (-T_0) \left(\sum_{j \in J} \phi_j\right), \tag{2.26}$$

where I and J are finite index sets with $J \subset I$. This has positive energy when J is a proper subset of I, reducing to (2.21) when J is the empty set. When $J = I$, on the other hand it reduces to (2.25), and has zero total energy.

More precisely, the total energy of such solitons does not exactly vanish, but for any finite θ, it is suppressed relative to normal D-brane energies by a factor of $1/\theta$. At finite θ, kinetic terms would contribute a small mass correction, as a result of which the energy of these solitons (following analogous arguments in ref. [17]) is generically $V(T_0) + \mathcal{O}(1/\theta)$. When we rescale coordinates to absorb a $\sqrt{\theta}$ in them, the total energy has a factor θ multiplying it, as in eq. (2.12). In these coordinates the mass of the above solitons is therefore finite, though the tensions of the standard stable and unstable D-branes of the theory are proportional to θ. If we revert to the original coordinates then the standard D-branes have finite tension and the solitons described above have total energies of order $1/\theta$.

The extension of the results in this section to unstable Dp-branes for various $p > 2$ (odd in type IIA and even in type IIB) is straightforward, for the case of noncommutativity only along two spatial directions. More interesting extensions to higher branes will be discussed in a subsequent section.

3. Non-commutative soliton as a smeared D-string

In this section we consider the D-brane charge associated with the above solitonic branes. Although the non-commutative solitons are not topological, and hence the exotic 0-branes of the previous section cannot carry a global RR charge, we can gain some insight into their nature by looking at what RR charge they carry locally.

As described for one special case in eq. (2.6) above, there is a Chern-Simons coupling on the non-BPS p-branes [11, 22, 25]

$$\frac{1}{2\,T_0}\int dT\wedge C_p\,, \tag{3.1}$$

where C_p is the Ramond-Ramond p-form, and T is the tachyon. In the limit when the noncommutativity parameter $\theta\to\infty$, we need only keep terms in the action which scale like θ (after coordinate redefinition). However, the Chern-Simons terms above, being topological, do not pick up any such factor, so in the $\theta\to\infty$ limit, they might appear to be irrelevant. This, however, is not the case. When T is a kink along one of the world-volume directions, we expect the solution to correspond to a (p -1)-brane, and therefore there must exist a coupling $\int C_p$ on its world-volume. The only source for such a coupling is the Chern-Simons coupling above, which can therefore not be neglected, even in the limit $\theta\to\infty$.

Now consider, as before, the unstable type-IIB D2-brane in a background $B_{\rm NS}$ field, and let us consider the tachyon background $T=T_0(1-f)$, where T_0 is the minimum of the tachyon potential, and f is, say, the gaussian solution ϕ_0. Since T_0 is constant, (3.1) yields

$$-\frac{1}{2\,T_0}\int T_0\,d\phi_0\wedge B=-\frac{1}{2}\int\phi_0'(r)\,B_{t\,\omega}\,dr\wedge dt\wedge d\omega\,, \tag{3.2}$$

where B is the Ramond-Ramond 2-form, so that the soliton locally carries D-string charge. In fact, since, $\int\phi_0'(r)\,dr=-\phi_0(0)=-2$, we get precisely $\int B_{t\,\omega}\,dt\wedge d\omega$, implying that the soliton carries unit D-string charge, and can in fact be interpreted as a D-string winding along the angular coordinate[4] ω. The function $f(r)$ can be interpreted as the distribution function for the D-string, so that our soliton describes a D-string smeared over the radial coordinate r, winding around the origin.

Such a D-string would not normally be stable, but would collapse by contracting to a point at the origin. This is equivalent to saying there is no global RR charge carried by this configuration. However, the above argument indicates that there is a local RR charge and enables an interpretation of the exotic decay product in terms of D-strings. To the extent that noncommutativity stabilizes some of the exotic 0-branes, which is discussed in a subsequent section, it will be equally true that such a smeared, winding D-string is stabilized.

[4]We will use ω to refer to the angular variable of the polar coordinates, reserving the more conventional θ for the noncommutativity parameter.

Note that if we choose $T = -T_0(1 - \phi_0)$, we get an anti-D-string instead. It is also easy to verify that, if we choose $f = \phi_n$, the soliton carries D-string charge $(-1)^n$. Again, if $f = \phi_0 + \phi_1$ (so that $T = T_0(1 - f)$ is still a classical solution) the total D-string charge vanishes. However, because the two functions ϕ_0 and ϕ_1 do not add to zero, we should really interpret this soliton as a dipole of D-strings smeared over the (r, ω) plane. This argument extends to the situation when we have $f = \sum \phi_i$ for some finite set of ϕ_i's.

Finally, for the solutions that are degenerate with the vacuum, for example $T(r) = T_0(1 - 2\,\phi_n(r))$, we find 2 units of D-string or anti-D-string charge locally, depending on whether n is even or odd.[5] For $T(r) = T_0(1 - 2\,\phi_0(r) - 2\,\phi_1(r))$, we again obtain a dipole of D-strings. The extension to solitons of the form (2.25) and (2.26) is obvious.

4. Stability of the decay products

Among the various classical solutions to the non-commutative tachyon theory, we found the class of solutions $T(r) = T_0(1 - 2\sum_{i\in I} \phi_i(r))$ with energy $V(T_0)$. This energy is negative, and is in fact just the same as we would have obtained by choosing the trivial classical solution (valid in both commutative and non-commutative cases) $T(r) = T_0$. Adding it to the tension $\mu_2^{(u)}$ of the unstable D2-brane, we get a total energy 0. Thus (in the limit of infinite noncommutativity) we find that an unstable D2-brane can decay into a configuration that, while quite different from the vacuum,[6] is nevertheless degenerate with it. In this situation one would expect the decay product to be stable.

One should, however, think of this as an approximation. As discussed in section 2, there will be corrections due to finiteness of the noncommutativity parameter θ. These corrections will tend to raise the energy of the decay product slightly above the vacuum, in which case it will be metastable. More information about the shape of the tachyon potential in string theory than is presently available would be needed to estimate the lifetime of this state.

The other solutions we described, such as the solution $T(r) = T_0(1 - \phi_n(r))$, are instead classically unstable. The easiest way to see this is to rewrite the total energy as follows[7]

$$V(T) + \mu_2^{(u)} = \widetilde{V}\,(T') \tag{4.1}$$

[5] From the formula we see that one of these two units of D-string charge is localised in the region where the field varies between T_0 and $-T_0$, while the other is localized in a region much closer to the origin, where the file goes from $-T_0$ to $-3T_0$. If we consider the boundary between these two regions, then the D-string charge jumps by a single unit across it. We are grateful to Ed Witten and Ashoke Sen for discussions on this point.

[6] The key difference from the vacuum is that this configuration carries local D-string charge, as described in the previous section.

[7] We wish to thank Rajesh Gopakumar for the following argument.

with $T' = T - T_0$. Then, $\widetilde{V}$ has degenerate minima at 0 and $-2\,T_0$, and a maximum at $-T_0$, and belongs to the general class of potentials discussed in [17]. From this, it is easy to see that the solution $T(r) = T_0(1 - \phi_n(r))$ corresponds to $T'(r) = -T_0\,\phi_n(r)$, which is unstable, as its energy can be decreased by scaling by a constant near unity. In other words, the energy of $T(r) = T_0(1 - \phi_n(r)) + \epsilon\,\phi_n(r)$, (for small ϵ) is *lower* than $T(r)$ by $\mathcal{O}(\epsilon^2)$.

These solutions have a finite energy above the vacuum and will decay into it classically. Hence they should be thought of as describing the decay of an unstable D2-brane into a kind of unstable D0-brane. The situation is somewhat reminiscent of the case of a brane-antibrane pair, which can decay into an unstable brane of one lower dimension by condensing an unstable tachyonic kink.

It is fascinating that, from the discussion in the previous section, the classically stable solitons all carry even D-string charge, while all the solitons with odd D-string charge are unstable. Note of course, that not all even charge solitons are stable. For example, the dipole $T_0(1 - \phi_0 - \phi_1)$, and the charge 2 soliton $T_0(1 - \phi_0 - \phi_2)$ are classically unstable.

5. Non-commutative decays of higher p-branes

We can generalize the argument of section 2 to higher brane solitons. Consider an unstable D3-brane of type IIA, with worldvolume along (x^0, x^1, x^2, x^3) in the presence of a constant background Neveu-Schwarz B-field along (x^1, x^2) directions. We expect the geometry along the (x^1, x^2) plane to become non commutative, while (x^0, x^3) remain commutative. We can now take the limit of large noncommutativity parameter $\theta = \theta_{12}$, and perform the appropriate rescalings of the coordinates. Now consider the tachyon background $T = K(x^3)(1 - f(x^1, x^2))$, where $K(x^3)$ is the kink along the commutative direction x_3, and f is one of the non-commutative solitons ϕ_n. Then from the coupling

$$\frac{1}{2\,T_0}\int dT \wedge C\,, \tag{5.1}$$

where C is the Ramond-Ramond 3-form, we get

$$\frac{1}{2\,T_0}\int_{D3}\Big((1-f)\,dK - K\,df\Big)\wedge C = \int_{D2}(1-f)\,C - \frac{1}{2\,T_0}\int_{D3} K\,df \wedge C\,, \tag{5.2}$$

where the first term on the r.h.s. is an integral over the D2-brane obtained from the kink K. In the absence of f, we should get therefore get a D2-brane. When $f = \phi_n$, however, from the term $\int_{D2} C(1-f)$ it appears that the D2-brane charge is altered by an "irrational" amount, since $\int_{D2} f(r)\,dx^1 dx^2 = 2\pi$, leading to a apparent contradiction. However, we should really examine the charge density, that is, the charge per unit D2-brane volume, and recall that the volume of the D2-brane is infinite. Thus the 1 in $(1-f)$ still contributes 1 unit of D-brane charge but since $\int_{D2} f(r)\,dx^1 dx^2$ is finite, it does not affect the charge density, so it can be ignored.

We also have a second term in (5.2):

$$-\frac{1}{2\,T_0}\int K\,df\wedge C\,. \tag{5.3}$$

As in the D2-brane case above, this gives

$$\frac{1}{T_0}\int K(x^3)\,C_{0\,\omega 3}\,dx^0\wedge d\omega\wedge dx^3\,. \tag{5.4}$$

Since the kink K goes from $-T_0$ to T_0 as x^3 goes from $-\infty$ to ∞, we see that we get zero total D2-brane charge. However, we really have a dipole of D2-branes, with the antibrane at $x^3<0$ (and the radial coordinate $r=\sqrt{(x^1)^2+(x^2)^2}$) and the brane at $x^3>0$ (and r).

Finally, from the coupling in the non-BPS D3-brane action:

$$\frac{1}{2\,T_0}\int dT\wedge A\wedge B_{\rm NS}\,, \tag{5.5}$$

where A is the Ramond-Ramond 1-form, and $B_{\rm NS}$ is the background Neveu-Schwarz B-field, we might expect the solitons to carry D0-brane charge. However, since the Neveu-Schwarz background is not quantized, we cannot in general expect integer (or even rational) D-brane charge from (5.5). We expect the resolution to this puzzle to be along the lines of [29], with the D-brane charge from (5.5) being cancelled by bulk terms.

The above exercise can be repeated with the metastable solutions of the type $K(1-2\,\phi_n(r))$ along the non-commutative directions, with similar conclusions.

6. Conclusions

It is pleasing that the field-theoretic study of non-commutative solitons initiated in ref. [17] has such an elegant application to the scalar field theory of tachyons on the world-volume of unstable D-branes in superstring theory. A more detailed understanding of the physical significance of this decay process, of the stability of the end products, and of modifications due to finite rather than infinite θ is clearly desirable.

In particular, consider the smeared D-string configurations that describe metastable decay products. It would be desirable to explain, from the spacetime (as opposed to brane worldvolume) point of view, why such configurations are rendered metastable by a suitable constant B-field.

It would be interesting to extend our results to Dp-branes of other dimensions, and to the non-commutative gauge theory that also resides on unstable D-branes. One could also consider the complex tachyon on a brane-antibrane pair, which couples

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to a worldvolume gauge field, unlike the real non-commutative tachyon studied in this paper. In such a case, one might combine non-commutative solitons with orthogonal tachyonic vortices in the non-commutative scalar-gauge theory that resides on brane-antibrane pairs. Finally, the physical effects of noncommutativity along, rather than transverse to, a topologically stable kink or vortex remain to be explored.

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